Pole/Zero Cancellations in Flexible Space Structures

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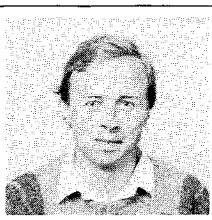
A practical objective in the control of flexible space structures is to minimize the effects of vibrational dynamics at certain specified points on a structure. State feedback can be used to address this question by creating closed-loop modes that are unobservable at these points and so do not contribute to the measured response. In the frequency domain, such modes correspond to pole/zero cancellations in the closed-loop system. This paper analyzes the problem of pole/zero cancellation in flexible structures, making full use of the second-order form of such systems. An explicit expression is derived for the unique state feedback gain with minimum norm that cancels all open-loop zeros. Furthermore, the properties of the residual poles that remain observable in the closed-loop system are studied, and their stability proven for the case of collocated sensors and actuators.

Introduction

A COMMON problem in the control of flexible space structures (FSS) is that of minimizing the effects of vibrational dynamics as measured at certain specified points on the structure, typically sensor locations or especially fragile components. This is of particular importance in view of the extremely low damping typical of FSS vibration modes (usually 0.5% of critical[1]), as the disturbance that results from any sensed flexible-body motion persists for a considerable time. For this reason, it is of great interest to investigate the use of feedback to make as many vibration modes as possible *unobservable*[2] at the specified points, therefore removing their effect totally from the closed-loop response at these points. This control strategy is the subject of this paper.

It is well known[3,4] that linear state feedback can be used to create unobservable closed-loop modes whereas the simpler output feedback cannot. Now, state feedback shifts the poles (eigenvalues) of the system but leaves its transmission zeros[2] (those frequencies at which a nonzero input can result in an identically zero sensed output) unchanged. Thus, an unobservable mode can be regarded in the frequency domain as being the result of shifting a closed-loop pole to the position of an open-loop transmission zero, so cancelling with it in the closed-loop transfer matrix. This pole/zero cancellation representation is particularly clear for single input/single output (SISO) systems, and has been shown[5] to apply equally well to multi-input/multi-output (MIMO) systems. The advantage of such a representation is that it specifies directly the number of modes that can be made unobservable by state feedback, as well as the values of the corresponding closed-loop poles. (These are simply given by the number and values of the transmission zeros of the system.) Note that the fixed closed-loop poles obtained when controlling a system by means of a linear optimal regulator, with no constraints on the allowable control gain, are precisely the transmission zeros of the open-loop system.[6] Thus, the fast output regulation characteristic of such a closed-loop system is, in fact, obtained by carrying out as many pole/zero cancellations as possible, i.e., by making the greatest possible number of modes unobservable.

The objective of this paper is to investigate the question of pole/zero cancellation in FSS, making full use of the second-order form of the equations of motion of structural dynamics. It will be shown that such an approach leads to new results for this problem. In particular, a direct characterization is derived for the unique state feedback that produces full pole/zero cancellation while minimizing the norm of control gain matrix needed. Furthermore, it is shown that the resulting *residual* poles, i.e., those that remain observable in the closed-loop system, have properties entirely analogous to those of the transmission zeros themselves. But the open-loop analysis of Ref. 7 proved that the zeros of any structure with compatible (physically collocated and coaxial) sensors and actuators must lie in a region of the left-half complex plane that is defined by the natural frequencies and damping ratios of the structure (see Fig. 1). The new closed-loop analysis presented here, therefore, implies that the residual poles of such a structure



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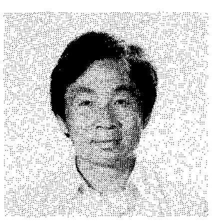

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must also lie in this region, and so are certainly stable. Furthermore, the specific values they take for any chosen distribution of sensor/actuator pairs can be computed by applying, without alteration, the algorithm developed in Ref. 8 for FSS transmission zeros computation. This is at least 15 times as fast as the general purpose zeros algorithm of Ref. 9, when applied to a lightly damped structure. It is interesting to note that knowledge of the pole/zero-cancelling state feedback is not needed for this calculation—the residual poles are obtained directly from the parameters of the open-loop system. Finally, if it is desired to shift any of the observable closed-loop poles away from these residual locations, the extra control required can also be characterized simply: it amounts to dynamic output feedback. These points will be illustrated by simple examples.

Poles and Transmission Zeros

Consider an n-mode linear model for the structural dynamics of a nongyroscopic, noncirculatory[15] FSS with m actuators and p sensors, not necessarily collocated. This can be written as

$$M\ddot{q} + C\dot{q} + Kq = Vu \tag{1a}$$

$$y = W_r\dot{q} + W_d q \tag{1b}$$

where q is the vector of generalized coordinates, u that of applied actuator inputs, and y that of sensor outputs. The mass, stiffness, and damping matrices of the structure satisfy $M = M^T > 0$, $K = K^T \geq 0$, and $C = C^T \geq 0$, respectively, and the control influence matrix V is of full column rank.

Taking the Laplace transform of Eqs. (1) yields the frequency domain polynomial matrix representation[10]

$$P(s)q(s) = Vu(s) \tag{2a}$$

$$y(s) = W(s)q(s) \tag{2b}$$

for the given FSS, where $P(s) = s^2M + sC + K$ and $W(s) = sW_r + W_d$. Note that $P(s)$ is symmetric, i.e., Eqs. (2) respects the special structure of the FSS equations of motion and so is particularly suited[7,8] to the study of such second-order systems. This is in contrast to the first-order state space representation $\dot{x} = Ax + Bu$, $y = Cx$ with $x = (\dot{q}^T, q^T)^T$, where A no longer preserves this useful symmetric structure.

The poles (or resonances) of the given system are those complex $s_i = \sigma + j\omega$ at which it is possible to obtain a nonzero output evolving with time as $\exp(s_i t)$, i.e., as $e^{\sigma t}\cos\omega t$, in response to an identically zero input. This occurs when the initial condition q_i on $q(t)$ is nonzero and chosen to satisfy $P(s_i)q_i = 0$, so clearly $P(s_i)$ must be of less than full rank for any pole. Conversely, the transmission zeros[2] are those complex s_i at which it is possible to apply a nonzero input and get an identically zero output in response, again for suitable initial conditions. Defining the system transfer matrix[10] $T(s)$ as the rational matrix satisfying $y(s) = T(s)u(s)$, the transmission zeros are clearly those s_i for which $T(s_i)$ is of less than full rank, and the desired output-nulling control is of the form $\exp(s_i t)u_i$, where $T(s_i)u_i = 0$. This is a natural generalization of the SISO case, where a transmission zero s_i makes the transfer function $T(s_i)$ zero, giving $y(s_i) = T(s_i)u_i = 0$ regardless of u_i. Note, however, that the transmission zeros of a MIMO system are *not* in general related to the frequencies that make zero any particular scalar transfer function $T_{ij}(s)$ between input j and output i. This fact, pointed out as long ago as 1974 in Ref. 11, is perhaps not yet fully appreciated.

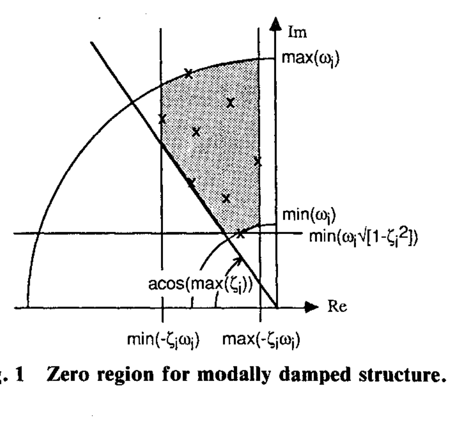

Fig. 1 Zero region for modally damped structure.

So long as its actuators and sensors have been positioned in such a way as to make it completely controllable and observable, that is, so that each mode can be both excited and sensed, an equivalent definition for the transmission zeros of this system is[2] those s_i for which the rank of the system matrix

$$S(s) = \begin{pmatrix} P(s) & V \\ -W(s) & 0 \end{pmatrix} \tag{3}$$

is reduced. This polynomial matrix condition is often more convenient to deal with than the original definition involving the rational $T(s)$ [$= W(s)P^{-1}(s)V$, by Eqs. (2)]. Furthermore, it illustrates the physical significance of the zero mode shapes of the structure. For, if $S(s_i)$ is of less than full rank, there exists a nonzero u_i and q_i for which

$$\begin{pmatrix} P(s_i) & V \\ -W(s_i) & 0 \end{pmatrix}\begin{pmatrix} q_i \\ -u_i \end{pmatrix} = 0 \tag{4}$$

and so comparison with Eqs. (2) shows that applying the nonzero input $\exp(s_i t)u_i$ to the system with generalized coordinate initial condition q_i gives rise to the identically zero output $y(s_i) = W(s_i)q_i$. Note that s_i and u_i are, of course, as given from $T(s_i)u_i = 0$, whereas the zero mode shape q_i can be regarded as the solution of a constrained modes problem,[12] with the constraint being simply that the sensor outputs remain identically zero. As a particularly simple special case, the zero modes of a rigid spacecraft with flexible appendages and a single sensor/actuator pair on the central body are the appendage-alone modes described in Ref. 13. Here, q_i corresponds to a natural vibration mode of the appendages in isolation, and the scalar u_i to the torque required to keep the central body at rest in the face of this resonance.

State Feedback and Pole/Zero Cancellations

A control law that has proved to be of great interest, and that is closely related to the preceding discussion, is linear state feedback.[2] In FSS terms, this becomes $u = F_r\dot{q} + F_d q + Gv$, a combination of rate and displacement feedback plus an external input term, or, in the frequency domain,

$$u(s) = F(s)q(s) + Gv(s) \tag{5}$$

where $F(s) = sF_r + F_d$. (Of course, in practical applications the entire state is not directly known, and so it must be reconstructed by means of an observer.[2]) Applying this control to the system described by Eqs. (2) clearly produces a closed-loop system with polynomial matrix representation

$$[P(s) - VF(s)]q(s) = VGv(s) \tag{6a}$$

$$y(s) = W(s)q(s) \tag{6b}$$

Now, it is well known[2] that, if the original system is completely controllable, all closed-loop poles can be arbitrarily assigned by suitable choice of $F(s)$; the transmission zeros, on the other hand, are invariant under state feedback. Furthermore, whereas controllability cannot be altered by state feedback, observability can; i.e., $F(s)$ can be chosen so as to make certain closed-loop modes unobservable. (The simpler output feedback control law, by contrast, cannot alter controllability or observability.) A practical reason for wishing to do this arises[4] if the open-loop system has a slowly decaying mode that prevents fast output regulation. It is likely to require less control effort to make this undesirable mode unobservable, and therefore no longer visible at the output, than it would to speed up its response substantially.

The mechanism by which state feedback creates unobservable modes can best be described as follows. Applying the pole characterization of the last section to Eqs. (6) shows that any closed-loop pole s_i and its corresponding mode shape q_i must satisfy $[P(s_i)-VF(s_i)]q_i=0$; if this mode is to be unobservable, it must also satisfy $y(s_i)=W(s_i)q_i=0$. But comparison with Eq. (4) shows that these two conditions are precisely the transmission zero defining relations [for state feedback with $v=0$, i.e., $u_i=F(s_i)q_i$], and so s_i and q_i must, in fact, be a transmission zero and its corresponding zero mode. This observation gives rise to the frequency domain notion that unobservable modes correspond to pole/zero cancellations in the closed-loop system: State feedback is considered to be chosen so as to shift an open-loop pole to the position of a transmission zero, cancelling with it in the closed-loop transfer matrix. This representation has the advantage that it specifies directly, from the zeros, the number of modes that can be made unobservable and the values of the corresponding closed-loop poles. It also illustrates the fundamental importance of the transmission zeros to a full description of the dynamics of any system.

It should be noted from the preceding equations that merely shifting a closed-loop pole to the position of a transmission zero is not in general sufficient to produce an unobservable mode: the associated mode shape must also be made equal to the corresponding zero mode. For single input systems, however, any pole and zero that are equal *are* guaranteed to result in an unobservable mode. These observations are perhaps most clearly illustrated from the transfer matrix $T(s)$. Consider first the SISO $T(s)=(s+1)/(s+2)$. Shifting the pole to $s=-1$ results in a closed-loop system with transfer matrix 1; i.e., a pole/zero cancellation has indeed occurred. On the other hand, $T(s)=\text{diag}\{1/s, (s+1)/(s+2)\}$ can be altered by state feedback to the closed-loop system with transfer matrix diag $\{1/(s+1), (s+1)/(s+2)\}$, which has a pole and transmission zero at $s=-1$ but no pole/zero cancellation. The explanation of this in terms of the polynomial matrix representation—namely, that the q_i satisfying $[P(s_i)-VF(s_i)]q_i=0$ for given s_i is unique only for $m=1$—is entirely analogous to the state space observation that the x_i satisfying $(A+BF)x_i=s_ix_i$ for given s_i is also only unique for $m=1$. Indeed, the freedom available to shape the closed-loop response of a multi-input system by state feedback can be regarded[4] as that of not only assigning the eigenvalues but also selecting each associated eigenvector x_i (or equivalently q_i) as any vector in the admissible subspace[14] corresponding to s_i. This close correspondence between the results obtained from the state space and polynomial matrix representations is a consequence of the fact that both are *internal* system descriptions involving a variable, x or q, which completely specifies the internal dynamics of the system. This is in contrast to the *external* description provided by $T(s)$, which only involves the system inputs and outputs. (See, for instance, Ref. 2 for further details.)

Flexible Space Structure Transmission Zero Properties

The special form of the equations of motion of vibrational dynamics can be shown to lead to interesting properties for the transmission zeros of large space structures. These properties are now briefly reviewed as background to the study of pole/zero cancellations in FSS given in the next section.

Let Z be any square root of the mass matrix M, i.e., any matrix satisfying $M=ZZ^T$. Two particular choices that are often proposed are[15] $Z=L$, where the lower triangular L is the Cholesky factor[16] of M, and [17,18] $Z=X\Lambda$, where M has eigen-decomposition

$$MX=X\Lambda^2 \tag{7}$$

with X orthogonal. (Another possibility is $Z=\Phi^{-T}$, where Φ is the modal transformation that gives $\Phi^TM\Phi=I$ and $\Phi^TK\Phi=\text{diag}(\omega_i^2)$; however, this is more complicated than required for the present application, as we do not need a diagonal stiffness matrix. Of course, if the given model is already in modal form, Z can be merely taken as the identity.) Defining the transformed $\bar{q}=Z^Tq$ and premultiplying Eq. (1a) by Z^{-1} then gives the modified system representation

$$\ddot{\bar{q}}+\bar{C}\dot{\bar{q}}+\bar{K}\bar{q}=\bar{V}u \tag{8a}$$

$$y=\bar{W}_r\dot{\bar{q}}+\bar{W}_d\bar{q} \tag{8b}$$

where $\bar{V}=Z^{-1}V$, $\bar{W}_r=W_rZ^{-T}$, and $\bar{W}_d=W_dZ^{-T}$. The transformed mass matrix is now equal to the identity, while $\bar{C}=Z^{-1}CZ^{-T}$ and $\bar{K}=Z^{-1}KZ^{-T}$ are both symmetric. We further operate on this model by means of the orthogonal transformation arising from the QR decomposition[16] of $\bar{V}$, i.e., the orthogonal Q for which $\bar{V}=QR$ with $R=(B^T,0)^T$, B upper triangular and nonsingular. Defining $\hat{q}=Q^T\bar{q}$ and premultiplying Eq. (8a) by Q^T then gives

$$\ddot{\hat{q}}+\hat{C}\dot{\hat{q}}+\hat{K}\hat{q}=Ru \tag{9a}$$

$$y=\hat{W}_r\dot{\hat{q}}+\hat{W}_d\hat{q} \tag{9b}$$

with $\hat{C}=Q^T\bar{C}Q$ and $\hat{K}=Q^T\bar{K}Q$ symmetric, and $\hat{W}_r=\bar{W}_rQ$, $\hat{W}_d=\bar{W}_dQ$.

The important features of this coordinate system are that the coefficient of $\ddot{\hat{q}}$ is just the identity matrix, while the control influence matrix R is of upper triangular form. As a consequence, the corresponding system matrix $\hat{S}(s)$ [see Eq. (3)] is also of highly structured form, therefore simplifying considerably a characterization of those s_i that reduce its rank, i.e., the system transmission zeros. The results that follow from this are particularly easy to interpret for the special case of compatible (physically collocated and coaxial) sensor/actuator pairs, so the bulk of the remainder of this paper will deal with this arrangement. We then have that $\hat{S}(s)$ is square (as $p=m$), and $\hat{W}_r$ and $\hat{W}_d$ are of similar form to R^T, i.e., $\hat{W}_r=(D_r,0)$ and $\hat{W}_d=(D_d,0)$. The corresponding system matrix, therefore, has the form

$$\hat{S}(s)=\left(\begin{array}{cc|c}\hat{P}_1(s) & \hat{P}_2(s) & B\\ \hat{P}_2^T(s) & \hat{P}_3(s) & 0\\ \hline -D(s) & 0 & 0\end{array}\right) \tag{10}$$

where $\hat{P}(s)=s^2I+s\hat{C}+\hat{K}$ has been partitioned conformably with B and $D(s)=sD_r+D_d$. By inspection, then, the transmission zeros of this system are those s_i for which either the $(m\times m)$ $D(s)$ is singular (the *sensor* zeros) or the $(n-m)\times(n-m)$ $\hat{P}_3(s)$ is singular (the *structural* zeros). It can be shown that the $2(n-m)$ structural zeros, which depend on the physical properties of the structure and the positions chosen for sensor/actuator pairs, always lie in the left half-plane. Furthermore, if as is often the case the structure is modally damped[12] with damping ratios $\{\zeta_i\}$, i.e., $\Phi^TM\Phi=I$, $\Phi^TC\Phi=\text{diag}(2\zeta_i\omega_i)$, and $\Phi^TK\Phi=\text{diag}(\omega_i^2)$, then the poles $-\zeta_i\omega_i\pm j\omega_i\sqrt{[1-\zeta_i^2]}$ of the system define a portion[7] of the left half-plane in which all these zeros must lie, regardless of the specific locations chosen for sensor/actuator pairs. This generic result, consisting of upper and/or lower bounds on the real and imaginary parts, moduli, and damping ratios of all structural zeros, is a consequence of the special form of the equations of motion of structural dynamics. It can be regarded as a generalization of the classical observation[13] that the zeros of a single input/single output undamped structure alternate with its poles along the imaginary axis. It admits a very simple graphical interpretation, as shown in Fig. 1 for an arbitrary distribution of poles x. Note that the highly structured form of Eq. (10) is also the basis of a recently developed algorithm[8] to compute the specific structural zeros arising from any particular choice for sensor/actuator locations. This is at least 60 times as fast as the general purpose zeros algorithm of Ref. 9 when applied to an undamped structure, and 15 times as fast for a lightly damped one. We now proceed to show that the transformed coordinate system of Eqs. (9) and (10) is equally valuable for the study of pole/zero cancellations in flexible structures, leading, for instance, to a simple explicit expression for the minimum-norm state feedback gain that achieves cancellation of all zeros.

Flexible Space Structure Pole/Zero Cancellations

A simple physical example should help to motivate the idea of pole/zero cancellation in flexible spacecraft. Consider the case of a spacecraft that is assumed for design purposes to behave as a rigid body, leading to the provision of precisely one sensor/actuator pair per rigid-body mode. If this vehicle in reality has certain flexible modes that must be included in any accurate model of its dynamics, then the discussion of the last section implies that this model will have precisely as many conjugate pairs of structural zeros, $(n-m)$, as flexible modes. Thus, full pole/zero cancellation could be used here to effectively remove all flexible-body effects from the sensor measurements of the closed-loop system; additional feedback could then be used to give the resulting "rigid-body" spacecraft any desired performance. It is interesting to note that the zero bounds illustrated by Fig. 1 guarantee that no structural zero will lie far from the open-loop poles, and so the feedback gain required to shift poles to all the zero locations is never likely to be prohibitively large.

In the coordinate system of Eqs. (9), state feedback becomes the control law

$$u(s) = \hat{F}(s)\hat{q}(s) + Gv(s) \tag{11}$$

where $\hat{F}(s) = F(s)Z^{-T}Q$ and $F(s)$, G and $v(s)$ are as given in Eq. (5). Application of this gives rise to the closed-loop polynomial matrix representation [c.f. Eqs. (6)] $\hat{P}_F(s)\hat{q}(s) = RGv(s)$, $y(s) = (D(s),\ \ 0)\hat{q}(s)$, where

$$\hat{P}_F(s) = \begin{pmatrix} [\hat{P}_1(s) - B\hat{F}_1(s)] & [\hat{P}_2(s) - B\hat{F}_2(s)] \\ \hat{P}_2^T(s) & \hat{P}_3(s) \end{pmatrix} \tag{12}$$

and $\hat{F}(s)$ has been partitioned conformably with $\hat{P}(s)$. If this feedback is to achieve full pole/zero cancellation, it must be chosen so that each transmission zero [each s_i for which $\hat{P}_3(s_i)$ is singular] is also a closed-loop pole, i.e., it also makes $\hat{P}_F(s_i)$ singular. This would obviously hold if $\hat{F}_2(s)$ were chosen to satisfy $\hat{P}_2(s) - B\hat{F}_2(s) = 0$, so reducing $\hat{P}_F(s)$ to block triangular form. We shall now show that this condition is, in fact, necessary as well as sufficient. For, the zero mode $\hat{q}_i = (\hat{q}_1^T, \hat{q}_2^T)^T$ associated with the ith unobservable pole s_i must satisfy

$$\left[\begin{array}{cc} [\hat{P}_1(s_i) - B\hat{F}_1(s_i)] & [\hat{P}_2(s_i) - B\hat{F}_2(s_i)] \\ \hat{P}_2^T(s_i) & \hat{P}_3(s_i) \\ \hline D(s_i) & 0 \end{array}\right] \begin{pmatrix} \hat{q}_1 \\ \hat{q}_2 \end{pmatrix} = 0 \tag{13}$$

so if the structural and sensor zeros are mutually distinct (which is certainly true for the common case of nonpositive real sensor zeros) $D(s_i)$ is guaranteed nonsingular, and so we must have $\hat{q}_1 = 0$. Thus, Eq. (13) reduces to the simpler condition that there exist an $\hat{F}_2(s)$ and a nonzero $\hat{q}_2$ for which

$$\begin{pmatrix} \hat{P}_2(s_i) - B\hat{F}_2(s_i) \\ \hat{P}_3(s_i) \end{pmatrix} \hat{q}_2 = 0 \tag{14}$$

Note that $\hat{F}_1(s)$ does not appear in this equation, a fact that will be readdressed below.

Now, for complete pole/zero cancellation, an expression of the form of Eq. (14) must apply for each of the $2(n-m)$ structural zeros, so each s_i that reduces the rank of $\hat{P}_3(s)$ must also reduce the rank of the composite matrix $([\hat{P}_2(s) - B\hat{F}_2(s)]^T,\ \hat{P}_3^T(s))^T$. Standard results[10] concerning the greatest common right divisors of two polynomial matrices therefore imply that $\hat{P}_2(s) - B\hat{F}_2(s) = H(s)\hat{P}_3(s)$ for some purely polynomial matrix $H(s)$. But the left-hand side of this equation has maximum degree 1, whereas $\hat{P}_3(s)$ has leading term s^2I, so $[\hat{P}_2(s) - B\hat{F}_2(s)]\hat{P}_3^{-1}(s) = H(s)$ is strictly proper,[10] only involving terms in s^{-1}, s^{-2}, etc. The only way $H(s)$ can be both strictly proper and polynomial, as required, is if it is, in fact, identically zero, i.e., if $\hat{F}_2(s) = s\hat{F}_{r2} + \hat{F}_{d2}$ is chosen to satisfy

$$B\hat{F}_2(s) = \hat{P}_2(s) \tag{15}$$

Note that the fact that B is nonsingular implies that such a choice is always possible.

We have, therefore, shown that the closed-loop denominator matrix $\hat{P}_F(s)$ has the block triangular form

$$\hat{P}_F(s) = \begin{pmatrix} [\hat{P}_1(s) - B\hat{F}_1(s)] & 0 \\ \hat{P}_2^T(s) & \hat{P}_3(s) \end{pmatrix} \tag{16}$$

for complete pole/zero cancellation, where $\hat{F}_1(s) = s\hat{F}_{r1} + \hat{F}_{d1}$ can take any desired value. The coordinate system of Eqs. (9) thus explicitly separates the fixed $[\hat{F}_2(s)]$ and arbitrary $[\hat{F}_1(s)]$ parts of all possible state feedback gains that give rise to complete pole/zero cancellation. The role of the arbitrary $\hat{F}_1(s)$ is to specify the values of those closed-loop poles that remain observable. As this feedback can be chosen to make the (1, 1) block of $\hat{P}_F(s)$ equal to $s^2I + s\tilde{C} + \tilde{K}$ for any freely chosen $\tilde{C}$ and $\tilde{K}$, it is always possible to give the observable poles any desired self-conjugate values. Note that this portion of the applied state feedback is of the form $\hat{F}_1(s)\hat{q}_1(s) = \hat{F}_1(s)D^{-1}(s)y(s)$, known as dynamic output feedback. This becomes simply a combination of outputs and their integrals if only rates are measured $[D(s) = sD_r]$, or outputs plus their derivatives if only displacements are measured $[D(s) = D_d]$. [The extremely simple constant output feedback control law $Gy(s)$ also provides some freedom to reassign the observable modes, although it is generally difficult to quantify this freedom precisely.]

A particularly interesting choice for zero-cancelling state feedback gain matrix is that which has the smallest possible norm, therefore reducing the control effort required. (But see the remarks in the next section concerning the plate example.) Clearly this is attained [in the coordinate system of Eqs. (9)] for the choice $\hat{F}_1(s) = 0$, giving rise to a closed-loop system with observable residual poles equal to those s_i for which the $(m \times m)$ $\hat{P}_1(s)$ is singular. Now, $\hat{P}_1(s)$ is a principal submatrix of $\hat{P}(s)$, just as the zero-defining $\hat{P}_3(s)$ is; therefore the bounds that were derived in Ref. 7 for the structural zeros also apply totally without alteration to the residual poles. In particular, these are consequently always stable, so no additional feedback $\hat{F}_1(s)$ is required to stabilize them. Furthermore, the residual poles of a modally damped structure must lie in the same region of the left half-plane, shown in Fig. 1, as do its transmission zeros. Similarly, the new efficient algorithm that was derived in Ref. 8 for computing FSS transmission zeros can be used without change to calculate the specific residual poles that result from particular choices for sensor/actuator positions. It is interesting to note that it is not necessary to calculate the zero-cancelling feedback $\hat{F}_2(s)$ first in order to be able to compute the residual poles; they are given directly from the open-loop $\hat{P}(s)$.

Although results as simple and explicit as those given by Eqs. (15) and (16) do not apply when sensors and actuators are not collocated, the basic principle of obtaining unobservable modes by carrying out pole/zero cancellations still holds for such systems. A practical point to bear in mind,[19] however, is that the transmission zeros of a structure with noncompatible sensors and actuators are not guaranteed to all lie in the left half-plane; it will generally be advisable to only cancel those that do, rather than creating unstable closed-loop poles. Once the set of zeros to be cancelled, $\{s_i\}$ say, has been determined, a state feedback $F(s)=sF_r+F_d$ that achieves this can be found by first solving Eq. (4) for $\{q_i\}$ and $\{u_i\}$ and then solving the underdetermined system of linear equations

$$U=(F_r,\ F_d)\ X \tag{17}$$

where U has columns $\{u_i\}$ and X columns $\{(s_i q_i^T,\ q_i^T)^T\}$. Note that, in contrast to the compatible case, the residual poles produced by the minimum-norm[16] solution to this equation are not necessarily stable; it may be necessary to select a higher-norm feedback solution in order to stabilize them.

The formulation for $F(s)$ based on Eqs. (4) and (17) is very similar to that used in Ref. 14 for robust eigenstructure assignment by state feedback, with the distinction between the two methods lying in the choice of eigenvector they make for each closed-loop pole. In Ref. 14, q_i is selected as that vector in the admissible subspace corresponding to s_i that makes this assigned pole as robust as possible, whereas in the present application the requirement that this mode be unobservable constrains q_i to be the (generally unique) zero mode shape corresponding to s_i. Thus, one method yields closed-loop modes that are robust but not decoupled from the measured outputs, whereas the other gives rise to unobservable modes but sacrifices some robustness in order to achieve this. So long as only left half-plane zeros are cancelled, though, the only consequence of this lack of robustness should be that some of the closed-loop modes that are ideally unobservable may become weakly observable, but still stable, as a result of system perturbations.

Finally, it is interesting to note that transmission zeros can equally well be defined for a set of fictitious sensors at various "points of interest" on the structure, rather than for the actual sensor locations. These points, generally not collocated with the actuators, could, for instance, be positions at which vibrational motion is particularly undesirable, e.g., weak structural elements or mounting points for sensitive instruments. If $W(s)$ in Eqs. (2) is defined to correspond to these pseudosensors, the method just outlined, based on Eqs. (4) and (17), can be applied unchanged to produce a closed-loop system with a set of modes that are unobservable at the points of interest. It should be noted here[2] that a system with more outputs than inputs will generically have no transmission zeros at all, thus preventing any pole/zero cancellation. This restriction, that there be only one point of interest per actuator, is clearly a physically reasonable one.

Examples

The preceding results will now be illustrated by applying them first to a system that is simple enough to permit hand computation of all quantities, so clearly demonstrating the processes involved, and then to a model more representative of a flexible spacecraft.

Three Mass-Spring-Dashpot System

The transmission zeros, residual poles, and minimum-norm zero-cancelling feedback will be computed for the simple three-mode vibrating system from Ref. 14, described by Fig. 2, with various choices for actuator and (real or fictitious) sensor locations. The masses and spring stiffnesses are selected to give $M=I_3$ and

$$K=\begin{pmatrix} 10 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{pmatrix}$$

respectively, and the dashpots are either absent or chosen so that

$$C=\begin{pmatrix} 2.5 & -0.5 & 0.0 \\ -0.5 & 2.5 & -2.0 \\ 0.0 & -2.0 & 2.0 \end{pmatrix}$$

giving open-loop poles as listed in Table 1.

Consider first the problem of cancelling the zeros of the system obtained by placing collocated actuators and displacement sensors at degrees of freedom one and three and setting $C=0$. Then, in Eqs. (2) we have

$$V^T=W(s)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

so (as $M=I_3$) the transformations leading to Eq. (9) are $Z=I_3$,

$$Q=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Equation (10), therefore, has components $B=D(s)=I_2$ and

$$\hat{P}(s)=\left(\begin{array}{cc|c} s^2+10 & 0 & -5 \\ 0 & s^2+20 & -20 \\ \hline -5 & -20 & s^2+25 \end{array}\right)$$

so, by inspection of $\hat{P}_3(s)=s^2+25$, the transmission zeros of this system are $\pm j5$; the residual poles [from $\hat{P}_1(s)$] are $\pm j\sqrt{10}=\pm j3.1623$ and $\pm j\sqrt{20}=\pm j4.4721$. Note that the zeros and residual poles all lie on the portion of the imaginary axis defined by the undamped version of Fig. 1, as predicted. Finally, Eq. (15) for this example yields the fixed component of all zero-cancelling state feedbacks as $\hat{F}_2(s)=\hat{P}_2(s)=(-5,\ -20)^T$, so the minimum-norm feedback [for $\hat{F}_1(s)=0$] becomes

$$F(s)=\begin{pmatrix} 0 & -5 & 0 \\ 0 & -20 & 0 \end{pmatrix}$$

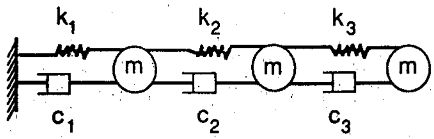

Fig. 2 Three mass-spring-dashpot system.

Table 1 Open-loop natural frequencies and damping ratios

		Damped case	
Mode, i	Undamped case, ω_i	ω_i	ζ_i
1	1.0344	1.0481	0.1442
2	3.2934	3.2517	0.3647
3	6.5638	6.5606	0.3296

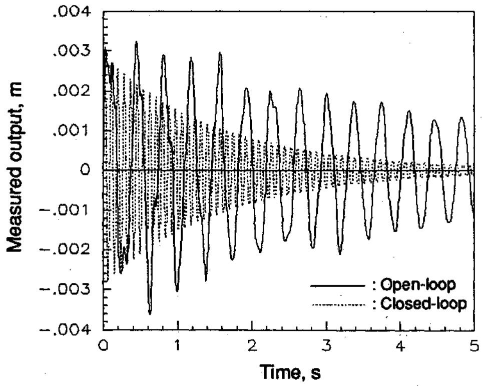

Fig. 3 Plate step response.

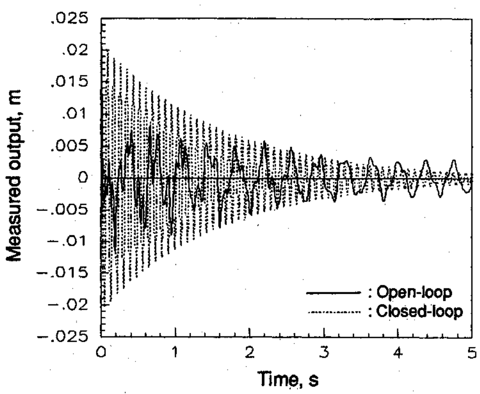

Fig. 4 Plate impulse response.

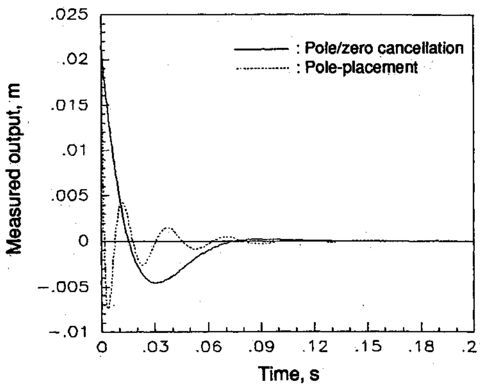

Fig. 5 Plate early impulse response.

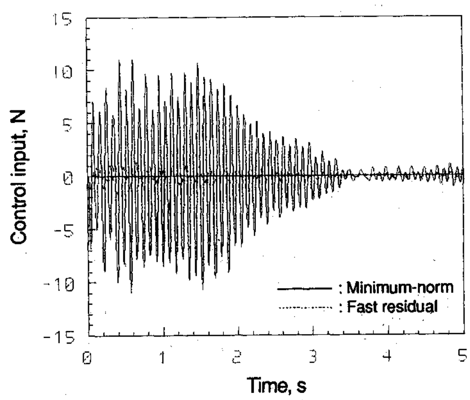

Fig. 6 Plate impulse response control forces.

in the original system coordinates. This corresponds to displacement feedback alone, as is to be expected for this undamped open-loop system.

Consider now the damped system with sensor/actuator locations as above. The analysis in this case proceeds very similarly to that just given, yielding that the transmission zero conjugate pair has modulus 4.9117 and damping ratio 0.2545; the residual poles have moduli $\sqrt{10} = 3.1623$ and $\sqrt{20} = 4.4721$ and damping ratios 0.3953 and 0.2236, respectively; and the minimum-norm zero-cancelling state feedback is

$$F(s) = \begin{pmatrix} 0 & -0.5s-5 & 0 \\ 0 & -2s-20 & 0 \end{pmatrix}$$

Note that one residual pole pair has a greater damping ratio than any open-loop pole, violating one of the bounds of Fig. 1. This is a consequence of the fact that C does not represent modal damping, so Fig. 1 does not apply here. All we can say[7] is that all zeros and residual poles must be stable, as they clearly are.

As a final case, we return to the undamped system, although now with a single actuator at the first degree of freedom. Suppose that the variable of interest is the relative displacement between the first and third masses. Equations (2) for this noncollocated fictitious sensor and actuator arrangement then has $V = (1, 0, 0)^T$ and $W(s) = (1, 0, -1)$. It can be shown from the resulting system matrix that this system has four transmission zeros, two at the origin and the conjugate pair $\pm j\sqrt{45} = \pm j6.7082$. Comparison with Table 1 shows that all these zeros violate the bounds that would have held had collocated sensors and actuators been used. Finally, application of Eqs. (4) and (17) shows that the zero-cancelling state feedback with minimum norm is $F(s) = (-5, 15, -5)$, in this case, purely displacement feedback as expected, and the resulting residual pole pair is $\pm j\sqrt{15} = \pm j3.8730$. Note that no closed-loop poles, observable or not, produced by F are unstable in this example, despite the noncollocated pseudosensor and actuator.

Uniform Plate

Simulation results will now be given to show the effects of pole/zero cancellations on the vertical steel plate used at DLR[20] as a laboratory test article for structural control experiments. This plate has horizontal length of 1.50 m, vertical length of 2.75 m, thickness of 2 mm, and isotropic material properties $E = 2.0 \times 10^{11}$ N/m^2, $\rho = 8.0 \times 10^3$ kg/m^3, and $\nu = 0.3$. For simplicity, it will be considered here to be simply supported along all four edges, leading[21] to a lowest natural frequency of 2.741 Hz and 10 modes with frequencies below 20 Hz. These modes are used to construct the truncated modal model used here, where each mode is assumed for illustrative purposes to have a damping ratio of 1%.

Consider a single linear sensor/actuator pair at a horizontal distance 0.6 m and vertical distance 1.2 m from the lower left tip of the plate, i.e., offset slightly from its central node point. There are then clearly $n - 1 = 9$ transmission zeros, the lowest at 4.321 Hz, and full pole/zero cancellation results in a closed-loop system with a single observable mode. If minimum-norm feedback is used, this residual mode can be shown to be at 11.607 Hz. It is clearly seen in the closed-loop response (dotted line) to a 1 N step input in v (taking $G = 1$), plotted in Fig. 3. (All units are m, N, s, as appropriate.) The improvement in the closed-loop performance over that of the open-loop system (solid line) is a consequence of the fact that the residual mode is over four times as fast as the slowest open-loop mode.

Similar comments apply to the responses to a 0.1 N impulse, plotted in Fig. 4, although the specific sensor/actuator location chosen for this example results in the output of the minimum-norm closed-loop system only becoming superior to that of the open-loop system after about 2 s. As noted previously, however, additional dynamic output feedback can be used to speed up the single observable closed-loop mode considerably, thus providing much faster output regulation. The solid line in Fig. 5 shows the result of using rate feedback to do this, with the real part of the observable pole chosen to be the quite large $-50\ \mathrm{s}^{-1}$. The measured output is now seen to be at rest after only 0.12 s, while Fig. 6 shows that the control effort

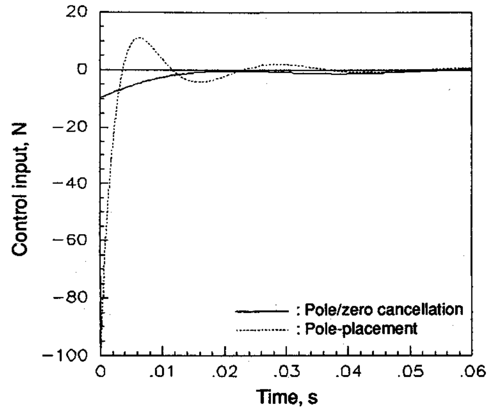

Fig. 7 Plate impulse response early control forces.

to achieve this (dotted line) is actually considerably less than that required (solid line) in the minimum-norm gain matrix case. This results from the fact that the displacements used to construct the feedback control force are now damped out much faster than before.

Finally, for comparative purposes, a standard pole-placement technique was used to produce a response roughly as fast as that just described by giving all closed-loop poles real parts of $-50\ s^{-1}$ (and imaginary parts equal to their open-loop values). The dotted line in Fig. 5 shows that this response is indeed about as fast as that obtained by the fast-residual pole/zero cancellation approach. The striking difference lies in the control inputs needed to achieve these results. As all closed-loop modes are observable in the pole-placement case, all 10 poles must be shifted, requiring (Fig. 7, dotted line) about ten times the peak control forces needed using pole/zero cancellation (solid line). Indeed, the norm of the pole-placement gain matrix is about 1.9×10^4 times as high as that of the alternative approach in this case!

Conclusions

This paper has analyzed the use of state feedback to make as many closed-loop vibration modes as possible unobservable at specified points on a flexible space structure, thus insuring that these modes do not contribute to the response at these points of interest. In the frequency domain, such modes correspond to pole/zero cancellations in the closed-loop system, so the special properties of the transmission zeros of flexible structures were shown to lead to interesting new results for this problem. In particular, an explicit expression was obtained for the unique state feedback gain with minimum norm that cancels all open-loop zeros of a structure with collocated sensors and actuators, and the resulting residual closed-loop poles were shown to lie in a region of the left half-plane that is defined by the natural frequencies and damping ratios of the structure. A question on which further work is planned is that of characterizing the robustness of the resulting closed-loop second-order system in the face of perturbations in the open-loop structural parameters. These points were illustrated by simple examples.

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